Math 2050, HW 3 (due: 24 Oct, before 23:59)

(1) If $x_1 < x_2$ are some real numbers and $x_n = \frac{1}{4}x_{n-1} + \frac{3}{4}x_{n-2}$ for $n > 2$. Show that $\{x_n\}_{n=1}^{\infty}$ is convergent and find the limit.

(2) Let $x_1 = 1$ and $x_{n+1} = 1 + \sqrt{x_n - 1}$ for all $n \in \mathbb{N}$. Show that the sequence is convergent and find the limit.

(3) Suppose all subsequence of (x_n) has a sub-sequence converging to 0. Show that $x_n \to 0$ as $n \to +\infty$.

(4) Suppose (x_n) is a sequence of positive real number. Show that

$$\limsup_{n\to+\infty} x_n^{1/n} \le \limsup_{n\to+\infty} \frac{x_{n+1}}{x_n}$$

provided that the limsup on the right hand side exists. Show that we cannot improve $\le$ to $=$ in general by providing an example.

(5) Suppose $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence such that x_n is an integer for any $n \in \mathbb{N}$. Show that there is N such that x_n is a constant for $n > N$.

(6) Let $p \in \mathbb{N}$ be fixed. Construct a example of (x_n) which is not cauchy but satisfies $|x_{n+p} - x_n| \to 0$ as $n \to +\infty$.